

At the boundaries of syntactic prehistory: metric and non-metric distances

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Giuseppe Longobardi, Luca Bortolussi, Andrea Sgarro

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Philosophical Transactions B, Royal Society (2021)

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Laura Franzoi, Andrea Sgarro, Anca Dinu, Liviu P. Dinu

*Random Steinhaus distances for robust syntax-based classification
of partially inconsistent linguistic data*

IPMU 2020, Lisbon (Pt)

Parametric Comparison Method PCM

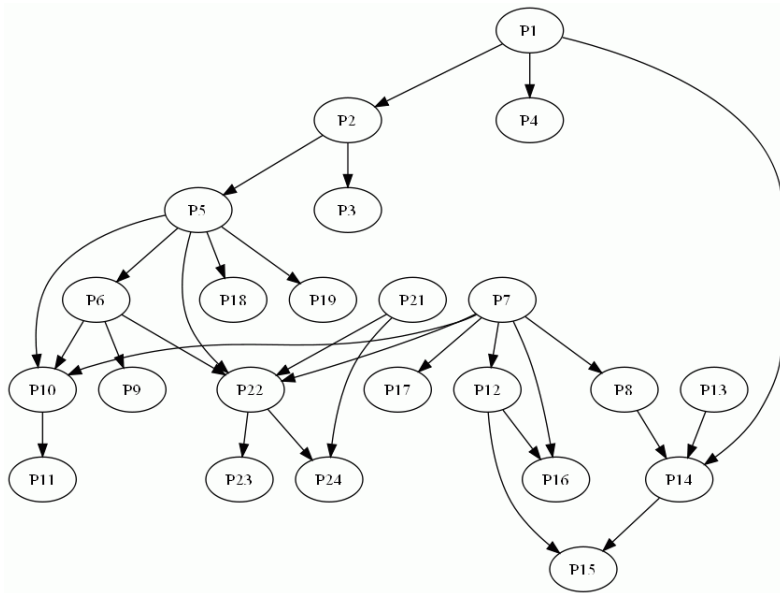
94 syntactic parameters, 58 languages from the Old World

possible languages

94 parameters as before, 5000 possible languages

results

controversial clusters such as Altaic (**Japanese, Korean, Mongolian, ...**) or Uralo-Altaic were significantly supported, while other possible macro-groupings as Indo-Uralic or Basque-Caucasian were not



Longobardi distances, Hamming-like and Jaccard-like

$L = 0 \mid 1 \mid * \mid 1 \mid 0 \mid 1$
 $\Lambda = 0 \mid 0 \mid * \mid * \mid 1 \mid 1$

$$\text{dist}_H(\Lambda, L) = \frac{\# \text{ bit differences}}{\text{"sound" bit length}} = \frac{2}{4} = \frac{1}{2}$$

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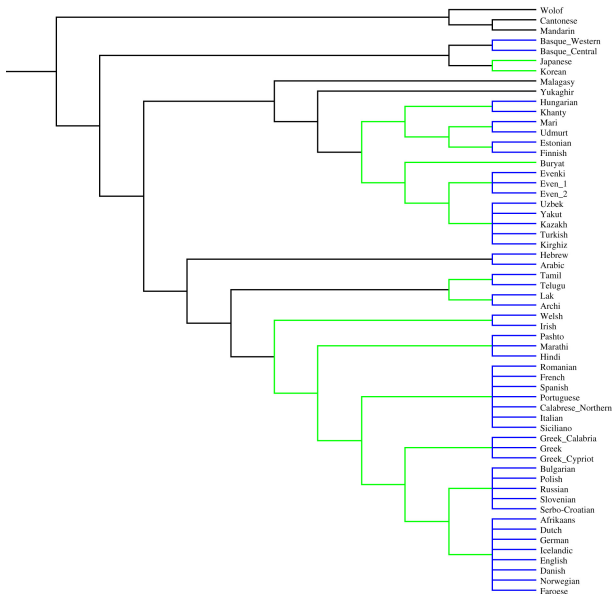
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both might violate the triangle inequality



what should a distance be?

at least...

- $d(x, y) \geq 0$
- $d(x, x) \leq \min [d(x, y), d(y, x)]$

(ordered) triangle inequality

$$d(x, y) \leq d(x, z) + d(z, y)$$

Steinhaus transform or **biotope transform** of the distance d :

$$S_d(x, y) \doteq \frac{2d(x, y)}{d(x, y) + d(x, z) + d(y, z)}$$

where:

- x, y, \dots are objects (possibly strings)
- $d(x, y)$ is their distance
- z is a fixed object called the **pivot** z

We'll have to generalize to several pivots
 $S_d(x, y)$ preserves metricity

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From (normalized) Hamming to Jaccard:
 the objects are n -length strings,
 the pivot $z = \underline{z}$ is the all-0 string

x, y strings of n logical values

$$d(x, y) = \sum_i [x_i \text{ AND } \neg y_i] \text{ OR } [\neg x_i \text{ AND } y_i]$$

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Solomon Marcus (1925-2016)

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Łukasiewicz: OR = $\min [(x + y), 1]$, AND = $\max [(x + y - 1), 0]$

taxicab or Minkowski or Łukasiewicz distance:

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$$* \implies \frac{1}{2}$$

$$d(\text{bit}, *) = d(*, \text{bit}) = \frac{1}{2}, \quad d(*, *) = 0$$

pivot of the Steinhaus transform: the "totally unsound" all-* sequence

consistency $\chi(x)$ of the string x : its taxicab distance from the all-* string

$$S_d(x, y) \doteq \frac{2d(x, y)}{d(x, y) + \chi(x) + \chi(y)}$$

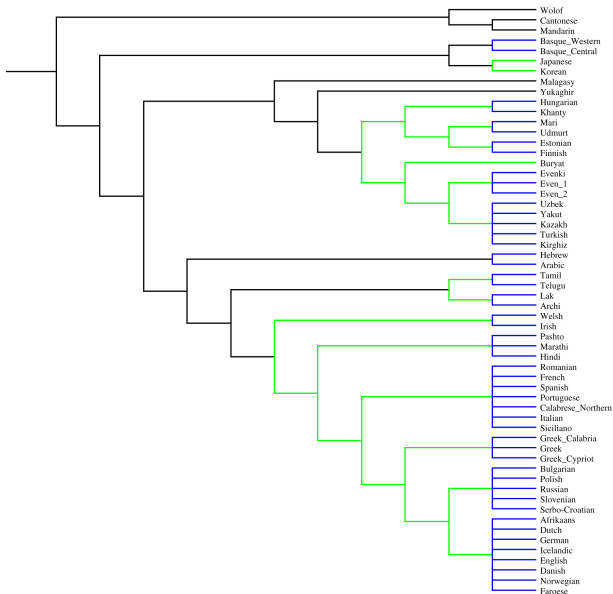
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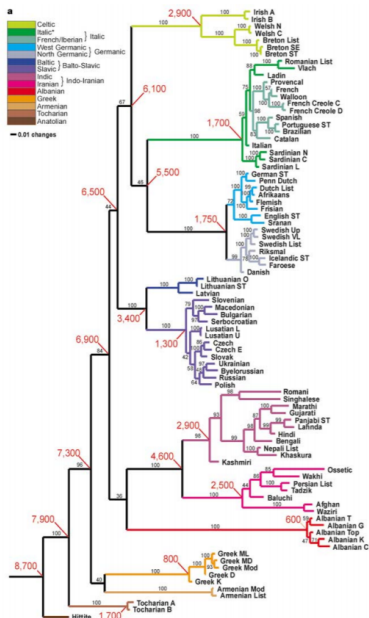
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weight $w(x)$ of the string x : its taxicab distance from the all-0 string

$$S_d(x, y) \doteq \frac{2d(x, y)}{d(x, y) + \min [\chi(x) + \chi(y), w(x) + w(y)]}$$





thanks, mulțumesc, grazie